MOODY'S ANALYTICS

Market Consistent Valuation

Moody's Analytics and Polish Insurance Association Event

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Agenda – Market Consistent Stochastic Modelling

- **Market Consistent Valuation** – Solvency II and Market-Consistent valuation of insurance assets and liabilities
- **Performing Market-Consistent Stochastic Simulation**
- **Things to note** when handling market-consistent simulations
- **Case Study** – Using market-consistent simulation in Monte-Carlo valuation of options and guarantee under Solvency II

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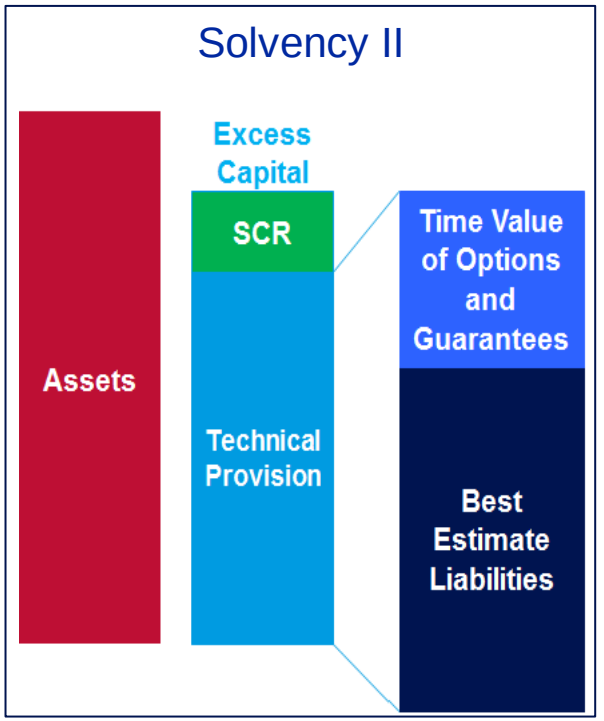
Market Consistent Valuation

Introduction

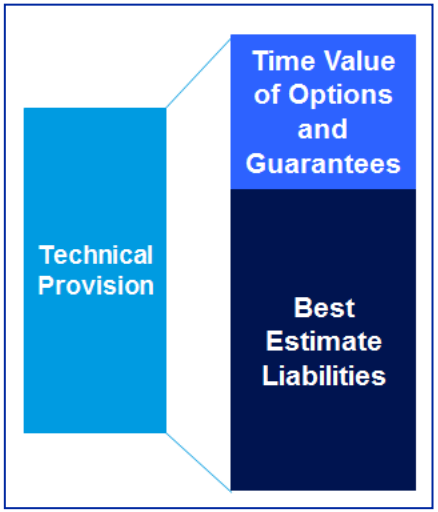
- Pre-Solvency II regime: Insurers are to define discount rates that are appropriate to their asset and liability exposures.
- Assets could be projected using their own risky returns
- Valuation does not converge to Market Value

- **Solvency II**: market-consistent valuation
- **All assets earn risk-free rate**
  - No arbitrage condition
  - Risk-free discounting and Risk-Neutral valuation
- Use ***Market Value*** wherever possible



Market Consistent Valuation

- Best estimate liability
  - Best estimate of insurance cashflows discounted at risk-free rate
- Time Value of Financial Options and Guarantees (TVoG)
  - Valued with "market-consistent" methods:
    1. Market Value of similar options/derivatives traded in the market
    2. Closed-form solutions
    3. Monte Carlo simulation using Market-Consistent stochastic scenarios



Example – Valuation of Simple Insurance Guarantee

Solvency II valuation of an insurance guarantee

» Intrinsic Value: Based on best-estimates

- If current value is larger than guaranteed value, "Solvency II Best Est. Liability" is 0 because you expect the guarantee to be worthless.
- But the true value of the guarantee is not 0 under Solvency II.

» Time Value of Guarantee: Based on option valuation techniques

Comparing the Solvency II TVoG Valuation Methods

Market Value

- Preferred method under Solvency II
- But least-used because it does not work if there are no “similar” derivatives liquidly traded in the market

Closed form solutions

- Approximations means there is an inevitable amount of under/over-estimation of TVoG
- Not all derivatives have a closed-form solution (e.g. American Options)

Stochastic Simulation (Monte Carlo Simulation)

- Can value complex options and guarantee (e.g. insurance profit-sharing products, value of dynamic lapse behaviours of policyholders)
- Much more accurate if models used are sufficiently sophisticated

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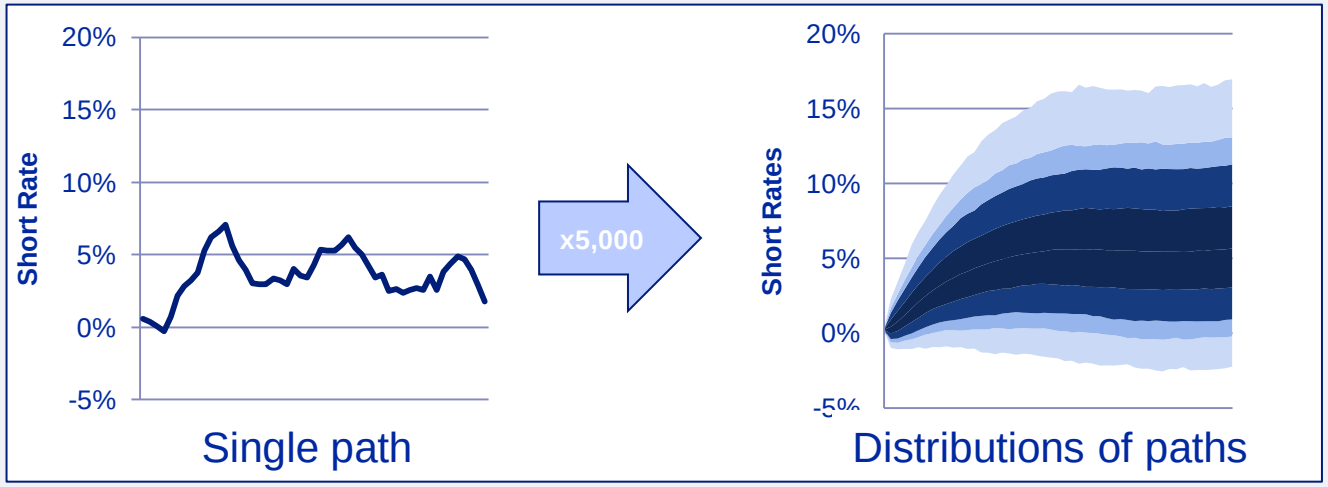
Performing Market Consistent Stochastic Simulations

What are Stochastic Simulations?

- Future is ***unknown***
- We may have ***expectations*** about the future but we are never ***certain*** about it
- Simulate ***many*** future scenarios based on stochastic models
- Use scenarios in ***Monte Carlo*** simulations by ALM systems
- ***Average*** of the ***Monte Carlo*** valuation converge to our expectation



Economic Scenario Generator

Definitions of different stochastic simulations

Market-Consistent

- All assets earn ***risk-free rate*** (same definition as Risk-Neutral)
- Monte-Carlo simulation ***replicates market-price***
- Distribution and statistics are market-implied

$\mathbb{Q}$

Real-World

- Risky assets earn ***risk-free rate*** PLUS ***risk-premium***
- Distribution and statistics are meaningful. One can set own assumptions about volatility and distributions of simulated rates.

$\mathbb{P}$

Performing Solvency II Market-Consistent Valuation using Stochastic Scenarios

☑ All assets earn risk-free rate

- Asset values discounted by risk-free cash account converges to market price

☑ Monte Carlo valuation replicates market price

- Risk-free discounted cashflow from derivatives converge to market traded derivative prices
- Market-consistent scenarios agrees with option implied volatility

? Quick Check: If I project future equity total returns and discount using risk-free rate will I calculate the same current value as the market value of equity?

- If not you have mispriced equity under Solvency II

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Things to Note When Handling Market Consistent Scenarios

Features of Market-Consistent Simulations

Because Market-Consistent valuation requires replication of ***Market Value***

- Market-Consistent Simulation results may not look reasonable from a "***Real-World***" perspective.
  - E.g. You may not expect ALL assets earn risk-free rate
  - E.g. You may not expect equity to be as volatile as what market traded option implies during the financial crisis in 2008
- But all such features are required to achieve a market-consistent valuation
- Otherwise the valuation will not fit Solvency II requirements
  - E.g. Monte Carlo price of a vanilla put option will be lower than the Market Value of the same put option – mispricing under Solvency II

How Do I Know My Scenarios Are Market-Consistent?

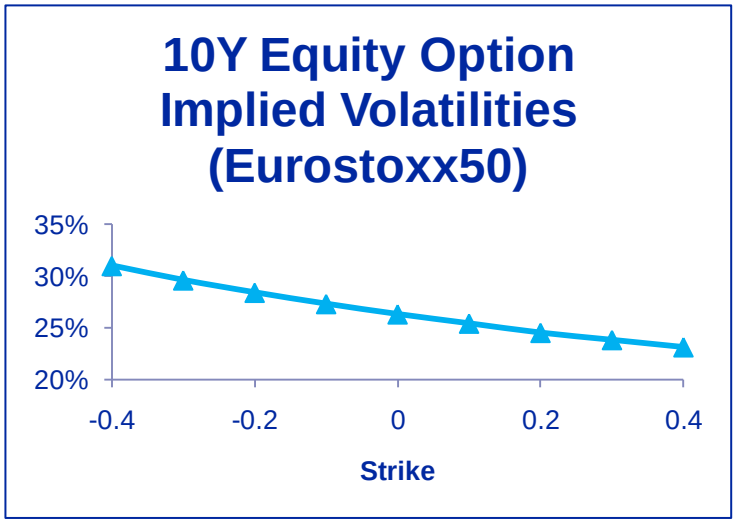
- Do all assets earn ***risk-free rate*** on average?
  - Check that the *average* risk-free discounted future price equals to the current price
  - This check is called a ***Martingale Test***
- Do Monte-Carlo option prices equal to Market option prices?
  - At different maturities?
  - At different strike prices?
  - With sophisticated models Monte-Carlo prices converge to Market option prices if Market Price exists

Pitfalls of using Closed-Form Solution

Why do we need Stochastic Simulation if we can use Black-Scholes?

Using Closed-Form Solutions are simpler and costs less to operate, but there are major pitfalls when it comes to insurance valuation.

- Solvency II requires
  - Market-consistent valuation of insurance options and guarantees
  - Modelling and valuation of dynamic behaviours
- Standard Black-Scholes mis-prices of out-the-money/in-the-money derivatives,
  - Causes issue under stresses
  - Leads to mis-calculation of SCR
- Closed-form solution does not exist for e.g.
  - Profit-sharing products
  - Path-dependent options and guarantees
  - Dynamic behaviours such as dynamic lapses



Using Market Consistent Scenarios

Inferring 99.5th Percentile from the Distributions

Can I use Market Consistent scenarios to determine the 99.5% percentile of my risk-capital calculation for SCR or ORSA?

- **No**. You need to use Real-World scenarios where assets earn ***risk-free rate PLUS risk-premium***
- To **project** insurance asset-liability forward into the future, whether one-year (SCR) or through business planning period (ORSA), Real-World simulation is required NOT Market-Consistent simulation

- [Rule of thumb]
  - To Value – Market-Consistent Simulations
  - To Project – Real-World Simulations

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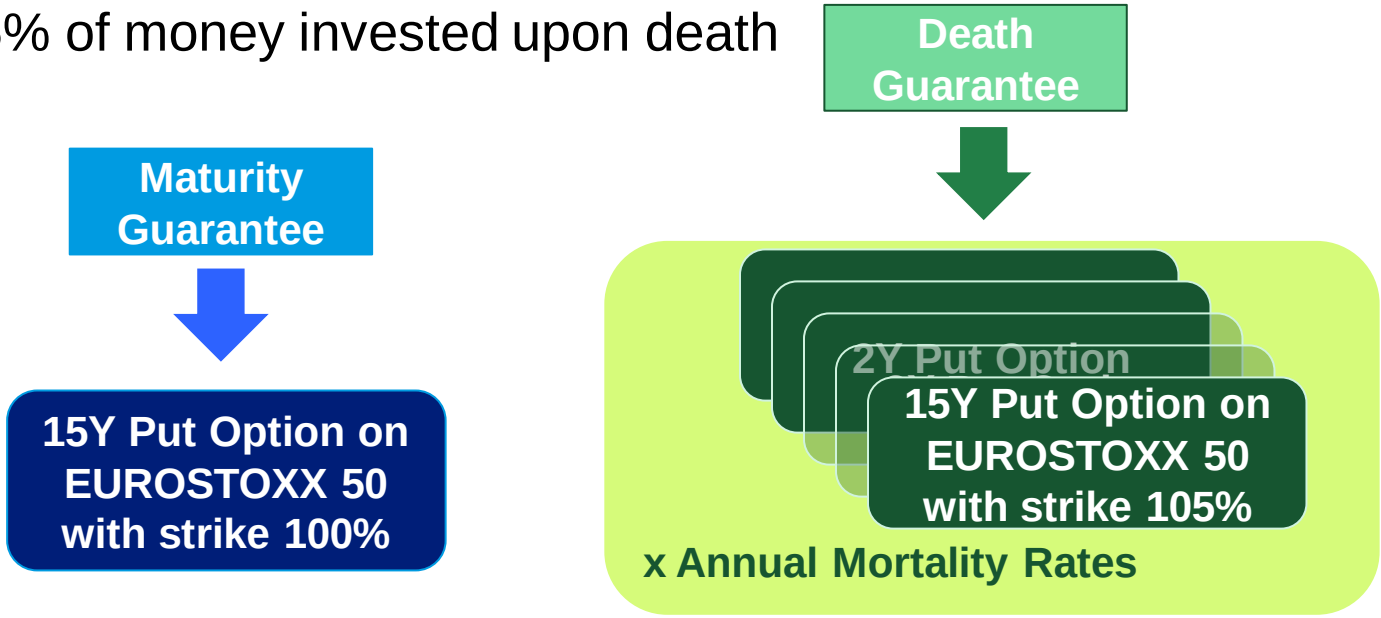
Case Study – Valuation of a Simple Insurance Guarantee

Case Study – Simple Insurance Guarantee Fund

- Single Lump Sum investment
- 15 Year Term, Launch Date 01/01/2008
- Underlying fund: EUROSTOXX50
- Guarantee 100% of money invested at maturity
- Guarantee 105% of money invested upon death



Base Scenario: No Stress

Valuation of Maturity Guarantee

- Valuation Time: 31/12/2012 (5 years after launch date)
- Remaining Maturity 10 Years
- EUROSTOXX 50 Total Return index at 71% of 2008 value
- Control Method: Market Price of 10-Year Put Option at -29% strike
- Method 1: Black-Scholes Formula; Method 2: Stochastic Simulation

- Black-Scholes under-estimating value of the guarantee by about 20%

Stress Scenario

What if I am only exposed at-the-money options and guarantee at the moment? Why should I worry about out-the-money or in-the-money valuations?

» Price of the underlying will change with time

» Similar situation arises when you perform a stress (e.g. -20% equity stress)

» Gets more complicated with Swaptions (for valuation of annuity guarantees such as Guaranteed Annuity Options)

- In swaptions, there are tenors, maturities and strikes, resulting in a 3-D volatility cube

From Simple Guarantee to Complex Liability

When there is no replicating derivatives

- Insurers can usually ***hedge*** simple guarantees such as the one in the case study with market-traded derivatives. Hence the value of the maturity guarantee is simply the value of a Put Option.
- However, it is very difficult to hedge ***path-dependent*** guarantees.
  - E.g. A fund which guarantee 1% year-on-year return.
  - Possible with advanced hedging techniques but the techniques themselves require stochastic simulations and frequent rebalancing
- Or it is plainly impossible to hedge: E.g. "***dynamic lapse behaviours***".
- To model and value these, ***advanced stochastic models*** have to be used.
  - Simple models perform poorly with skewed implied volatilities
  - To capture behaviours beyond *Normal* distribution requires sophisticated models.

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Question and Answer

Summary

Solvency II Market Consistent Valuation of ***Insurance Liability*** in general

» Discount using risk-free rate

Solvency II Market-Consistent Valuation of ***Time Value of Options and Guarantee***

1. **Market Value** of Replicating Derivatives
   - Most insurance options and guarantees cannot be replicated by market derivatives
2. **Closed-Form Solution**
   - Close-form solution does not exist for complex insurance cashflows
3. **Market-Consistent Stochastic Simulation**
   - Takes scenarios by an Economic Scenario Generator (ESG)
   - Covers complex options, path-dependent guarantees and dynamic behaviours

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